THE PLEASURE OF MATHS - Pierre Deligne

Pierre Deligne: “In my view, resolving problems by methods which, at first sight, seem to have nothing to do with the problem at hand, is one of the great joys that mathematics offers to the professional mathematician.”

He has received some of the world’s most prestigious awards, including the Fields Medal, and rubs shoulders with the finest minds of our time. He is one of the greatest mathematicians of the century, but what strikes one most about Belgian-born Pierre Deligne is his modesty and affability. He is a professor at the Princeton Institute for Advanced Study in the United States, and his talent has just earned him the Wolf Prize in mathematics.

It is 24 May 2008 and the evening is balmy. While the guests sip their cocktails, two figures slip quietly onto the restaurant terrace. Pierre Deligne and his wife have withdrawn from the world for a moment to admire the lights mingled with the stars, and Jerusalem in the distance stretched out at the foot of mount scopus. Tomorrow, at the Knesset, the Israeli parliament, Pierre Deligne will receive the wolf prize, along with David Mumford and Philip Griffiths. All three are Leaders of algebraic geometry, a discipline reputed for its beauty and difficulty.

Mathematics is second nature to Deligne. “When I was at primary school, I remember there was an experiment to measure the surface of a sphere compared with that of a disk.

The teacher wound a string around the entire sphere and then compared the length of the string with the same string coiled flat on the disk. It was obvious to me that this demonstration was nothing of the kind! You can’t measure a surface area in terms of length … but I still found it interesting from a mathematical standpoint.”

When Pierre Deligne was around 12years of age, he would read his brother’s university maths handbooks and pester him for explanations. “I was astonished to learn that third degree equations could be resolved by extracting the square roots and cubic roots using fairly complicated formulae, and that this also worked for fourth degree equations… but not for those of the fifth degree. I found that extraordinary!”

His interest prompted Mr Nijs, the father of a fellow scout and a maths teacher, to give him a book that people would normally never dream of offering a 14-year-old child: Bourbaki’s Theorie des ensembles (Theory of sets)! This highly abstract treatise constructs a whole field of mathematics step by step from the very beginning. “It took me a year to read it, complete the exercises and really understand it, because it is understanding that counts. It was worth every minute! It taught me what the ideal of rigour in mathematics was. And once you have understood this ideal, you feel much freer to depart from it, since you know that you can always return to that ideal.”

At the universite Libre de Bruxelles – ULB (BE) where he had been invited to attend a postgraduate seminar at the tender age of 16, everyone was ‘in the know’. Later, one of his professors, jacquesTits, who to create the mathematical theory of buildings, steered him towards the famous IHES in Paris (the Institute des Hautes Etudes Scientifiques – Institute of Advanced Scientific Studies), where a team of brilliant mathematicians, led by Alexander Grothendieck, was overhauling algebraic geometry to turn it into a monument to human thought.

Algebraic geometry

It has taken centuries for us to come to the realisation that, much of the time, geometry and algebra express the same underlying reality in two different languages. Most of Pierre Deligne’s work has focused on algebraic geometry. It is a discipline with a finger in every pie, mainly because it pops up naturally anywhere that polynomials are found.

“To give some idea of what this means, let us take the graphs so familiar to us, where a point in a plane, plotted according to two perpendicular axes, is identified by two numbers, and where a curve represents a relationship between these numbers, for example changes in magnitude over time (like temperatures or stock market prices). The use of these Cartesian coordinates dates back to Fermat and Descartes. Thus, point (x,y) where x and y verify an equation of the first degree, for example $2x + 3y-6 = 0$,are the points on a straight line. Hence the term ‘linear equation’.Similarly, a point in space, plotted according to three perpendicular axes is identified by three numbers, and a plane corresponds to a linear relationship (of the first degree) between these numbers. This is the start of a dictionary to correlate geometric figures with algebraic expressions. It possible to apply to one group the very different intuition we have of the other group, and to extend these intuitions to deepen our understanding.”

This means that there is a two-away relationship between algebra and geometry. For instance, a complex multidimensional figure that is too difficult to be imagined can be defined algebraically, and vice versa. So work in one field can advance the other.

Fame for solving conjectures

In 1970, when he was only 25, Pierre Deligne became a permanent member of the IHES where, building on the work of his mentor Alexander Grothendieck, his work took off spectacularly in 1973 and culminated in his proving a series of very difficult conjectures first proposed by Andre Weil in 1949, mainly concerning analogues of the Riemann hypothesis on the zeta function.

Part of the creative work of maths researchers is to put forward theorms which, until they are proven, remain mere conjectures. Although conjectures can to some degree be intuited by deductive and inductive reasoning, they must be proven with absolute rigour. A simple conjecture may require centuries of effort to find a counter-example to either shatter or prove it. For instance, Fermat’s famous 17th century conjecture was not proven by Andrew Wiles until 1994.

According to Weil’s three conjectures, it ought to have been possible to obtain results about the number of integer solutions to polynomial equations over finite fields using intuition on how algebraic topology should apply in this novel situation. Although Weil was in no doubt, he was never able to actually prove his conjectures.

By combining a new branch of topology developed by Grothendieck (etale cohomology) with partial results obtained by Robert Rankin on an old Ramanujan conjecture, Which was apparently totally unconnected, Deligne succeeded in proving the Weil conjectures in 1973. His supremely elegant result not only resolved a major problem of mathematics, it also proved that disparate subjects were in fact connected. This catapulted Deligne to instant fame. The International Mathematical Union awarded him the fields Medal in 1978 in recognition of his contribution to advancing the unification of algebraic geometry and number theory.

Pierre Deligne has himself put forward conjectures that have been proven by other researchers, which shed light on various fields of mathematics, or even physics, such as string theory.

IAS in Princeton

In 1980, Deligne married Elena Vladimirovna Alexeeva, with whom he had his two children. Later he took his family to the United States, where he continued his research at the Institute for Advanced Study in Princeton. Deligne is excited by the idea of uniting apparently disparate fields – group theory, topology, Grothendieck's theory of motives – which calls for a broad mathematical culture, an agile mind and outstanding inventiveness.

"In my opinon, seeing the ways in which different people present points of view, and these opposing views yield flashes of insight! I remember that when my children had a problem with maths, it really annoyed them if I explained the problem from five different points if view when all they wanted was the ONE correct answer!"

Pierre Deligne identifies closely with the Russian school of mathematics which he first came across in 1971, and was one of the most brilliant in the world in the 20th century but is now threatened by the brain drain. In order to help young Russian mathematicians to work in their homeland, he used prize money from the Balzan Prize for mathematics, which he won in 2004, to create the Pierre Deligne Contest, awarding three –year grants administered by the Independent University of Moscow.

He enjoys simple pleasures, such as gardening, walking and cycling. So, Pierre Diligne, why do you do maths? "Because it's so beautiful of course!"